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An induced OWA operator in coal mine safety evaluation

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ABSTRACT

Aggregation operators are crucial to decision-makers when they make decisions. The Ordered Weighted Aggregation (OWA) is the most common operator to aggregate the arguments that are the exact numerical values. However, the decision-makers may have vague knowledge about the decision information, and can't estimate their decision information with exact numerical values. Later, some new families of OWA operators appeared, *e.g.*, a Linguistic Ordered Weighted Geometric Averaging (LOWGA) operator. Inspired by LOWGA, we propose an induced LOWGA operator, and then study some desirable properties of the operator. Based on the operator, we propose a decision-making method for coal mine safety evaluation. Safety is not only an eternal topic in coal mining but also is fundamental in the process of coal mine production, so it is important to establish a scientifically justified evaluation system and aggregate the decision information with the linguistic values. In this paper, the method is straightforward and has no loss of information, because we not only consider the weight of the factors affecting coal mine safety, but also take the ordered position of the factors in aggregation process. Both the theoretical analysis and the comparative results show that the method can better reflect the real situations in coal mine safety evaluation.

1. Introduction

With the development of computer science and technology, huge amount of data are produced from various sources. It is crucial for a decision-maker to use these data efficiently making a decision. At present, many kinds of decision-making techniques have been proposed [11–13]. One of the most interesting and complex decision-making issues is multiple attribute decision making. Two principal problems involved in solving a multiple attribute decision making are to aggregate attribute value and to weigh the relative importance of the objectives.

How to aggregate and process the given data effectively is a very important issue in decision-making problems. The scholars at home and abroad have put a great deal of research on the aggregation operators. These operators range from the simple arithmetic mean to fuzzy-oriented ones like minimum/maximum and t-norm/t-conorm. In addition, Yager [19] introduced a parameterized mean-like aggregation operator, an ordered weighted aggregation (OWA) operator. Essentially, by selecting an appropriate weight vector, the OWA operator can reflect the uncertain nature of human judgment with the ability to generate an aggregating result lying between two extremes of minimum and maximum. Inspired by OWA operator, some new families and applications of OWA operators were introduced [1,2,5,15,17]. The OWA operators have been applied in different areas [20,21] such as economics, management, the military, safety evaluation, etc.

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An evaluation set must be given in advance from which the decision-maker can choose to evaluate the objectives and weigh the importance of the objectives. The evaluation set maybe a numerical or natural language terms set. But in real decision-making problems, most of information can be qualitative in nature, for instance, with vague or imprecise knowledge. It would be a more realistic approach in qualitative setting to use linguistic assessments instead of numerical values. How to aggregate and process the linguistic terms is a hot issue in decision-making problems. There are some scholars who have studied the linguistic terms [3,4,6,7,10,14,22] and have already made certain achievements.

However, few of these operators can both consider the weight associated to the operators and the weight of the attributes synchronously. And few of scholars study the application of aggregation operators in coal mine safety evaluation. In this paper we propose an Induced Linguistic Ordered Weighted Geometric Averaging (ILOWGA) operator, which is more rational because we consider not only the importance of the ordered position of the argument but also the given argument itself, furthermore we develop a decision-making method for coal mine safety evaluation with linguistic values. In the coal mine safety evaluation, the decision-makers usually have vague knowledge about the decision information, and can't estimate their decision information with exact numerical values. It is more suitable to provide their preferences by means of linguistic variables rather than numerical ones. For example, the experts usually evaluate geological condition of coal mine with a linguistic term set {very active, active, fair, steady, very steady}.

The rest of this paper is organized as follows. Section 2 introduces some aggregation operators and some desirable properties. Section 3 proposes an Induced Linguistic Ordered Weighted Geometric Averaging (ILOWGA) operator and studies some desirable properties. Section 4 establishes a coal mine safety evaluation indicator system and develops a decision-making method for coal mine safety evaluation, which is rational and has no loss of information. In this section, the proposed method turns out to be feasible and practical in coal mine safety evaluation through its application. Section 5 concludes the paper.

2. Preliminary

OWA operator can be expressed as following [19]: OWA: $R^n \to R$, if $\mathrm{OWA}_w(a_1,\ a_2,\ldots,a_n) = \sum_{j=1}^{n} w_j b_j$, where $w = (w_1, w_2, \ldots, w_n)^T$ is the associated weighting vector, with $w_j \in [0,1]$ such that $\sum_{j=1}^{n} w_j = 1$, and b_j is the jth largest element in the set $\{a_1, a_2, \ldots, a_n\}$, then the function OWA is called the ordered OWA operator of dimension n.

The fundamental aspect of the OWA operator is the re-ordering step. In particular, an argument a_i is not associated with a particular w_i, but rather a weighted w_i is associated with a particular ordered position i of the arguments (therefore, the weighting vector w is also called the position vector).

The OWA operator has the following properties [19]:

1) (Commutativity) Let $(a_1, a_2, \ldots, a_n)$ be a collection of arguments, and $(a'_1, a'_2, \ldots, a'_n)$ be any permutation of $(a_1, a_2, \ldots, a_n)$. Then $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = \mathrm{OWA}_w(a'_1, a'_2, \ldots, a'_n)$.

2) (Idempotency) Let $(a_1, a_2, \ldots, a_n)$ be a collection of arguments if $a_i = a$, for any i. Then $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = a$.

3) (Monotonicity) Let $(a_1, a_2, \ldots, a_n)$ and $(b_1, b_2, \ldots, b_n)$ be two collections of arguments, if $a_i \leqslant b_i$, for any i. Then $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) \leqslant \mathrm{OWA}_w(b_1, b_2, \ldots, b_n)$.

4) (Bounded) The OWA operator lies between the max and min operators: $\min(a_i) \leqslant \mathrm{OWA}_w(a_1, a_2, \ldots, a_n) \leqslant \max(a_i)$.

5) If $w = \{\frac{1}{n}, \frac{1}{n}, \ldots, \frac{1}{n},\}^T$, the OWA operator is reduced to the arithmetic average operator: $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = \frac{1}{n}\sum_{i=1}^{n} a_i$.

6) If $w = (1, 0, \ldots, 0)^T$, the OWA operator is reduced to the max operator: $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = \max(a_i)$.

7) If $w = (0, 0, \ldots, 1)^T$, the OWA operator is reduced to the min operator: $\mathrm{OWA}_w(a_1, a_2, \ldots, a_n) = \min(a_i)$.

Definition 1. (See [18].) An Ordered Weighted Geometric Averaging (OWGA) operator of dimension n is a mapping $g : R^{+^n} \to R^+$, that has associated with it a weighting vector $w = (w_1, w_2, \ldots, w_n)^T$, with $w_i \in [0,1]$ and $\sum_{i=1}^{n} w_i = 1$, such that:

$$g(a_1, a_2, \ldots, a_n) = \prod_{j=1}^{n} b_j^{w_j},$$

where b_j is the jth largest element in the set $\{a_1, a_2, \ldots, a_n\}$.

Clearly, the elements b_j $(j = 1, 2, \ldots, n)$ in Definition 1 are arranged in descending order:

$$b_1 \geqslant b_2 \geqslant \cdots \geqslant b_n.$$

The OWA and the OWGA operators have only been used in situations in which the input arguments are the exact values. However, judgements of people depend on personal psychological aspects such as experience, learning, situation, state of mind, and so forth. It is more suitable to provide their preferences by means of linguistic variables rather than numerical ones (for example when evaluating the comfort or design of a car, terms like good, fair, poor can be used). In the following, we shall introduce some aggregation operators, which can be used to accommodate the situations where the input arguments are linguistic variables.

Let $S = \{s_i\}$ $(i = 1, \ldots, t)$ be a finite and totally ordered discrete term set. Any label, s_i, represents a possible value for a linguistic variable, and it must have the following characteristics [8]:

(1) The set is ordered: $s_i \geqslant s_j$ if $i \geqslant j$;
(2) There is the negation operator: $neg(s_i) = s_j$ such that $j = t + 1 - i$;
(3) Max operator: $\max(s_i, s_j) = s_i$ if $s_i \geqslant s_j$;
(4) Min operator: $\min(s_i, s_j) = s_i$ if $s_i \leqslant s_j$.

For example, S can be defined so long as its elements are uniformly distributed on a scale on which a total order is defined:

$$S = \{s_1 = \text{very poor},\ s_2 = \text{poor},\ s_3 = \text{fair},\ s_4 = \text{good},\ s_5 = \text{very good}\}.$$

To preserve all the given information, Xu extended the discrete term set S to a continuous linguistic term set $\bar{S} = \{s_\alpha \mid s_{1/t} < s_\alpha \leqslant s_t, \alpha \in [1/t, t]\}$, where, if $s_\alpha \in S$, then s_α is called the original linguistic term, otherwise, is called the virtual linguistic term. Consider any two linguistic terms $s_\alpha, s_\beta \in \bar{S}$, and $\mu, \mu_1, \mu_2 \in [0,1]$, some operational laws are defined as follows [16]:

(1) $\mu s_\alpha = s_{\mu\alpha}$;
(2) $(s_\alpha)^\mu = s_{\alpha^\mu}$;
(3) $(s_\alpha)^{\mu_1} \otimes (s_\alpha)^{\mu_2} = (s_\alpha)^{\mu_1+\mu_2}$;
(4) $(s_\alpha \otimes s_\beta)^\mu = (s_\alpha)^\mu \otimes (s_\beta)^\mu$;
(5) $s_\alpha \otimes s_\beta = s_\beta \otimes s_\alpha = s_{\alpha\beta}$.

Definition 2. (See [16].) Let LWGA: $\bar{S}^n \to \bar{S}$, if

$$\begin{aligned}\text{LWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\alpha_1})^{w_1} \otimes (s_{\alpha_2})^{w_2} \otimes \cdots \otimes (s_{\alpha_n})^{w_n} \\ &= (s_{\alpha_1^{w_1}}) \otimes (s_{\alpha_2^{w_2}}) \otimes \cdots \otimes (s_{\alpha_n^{w_n}}) = s_\alpha,\end{aligned} \tag{1}$$

where $\alpha = \prod_{j=1}^n \alpha_j^{w_j}$, $w = (w_1, w_2, \ldots, w_n)^T$ is the exponential weighting vector of the s_{α_j}, and $w_j \in [0,1]$, $\sum_{j=1}^n w_j = 1$, $s_{\alpha_j} \in \bar{S}$, then LWGA is called the Linguistic Weighted Geometric Averaging (LWGA) operator.

Especially, if $w = (1/n, 1/n, \ldots, 1/n)^T$, then LWGA is called the Linguistic Geometric Averaging (LGA) operator.

Definition 3. (See [16].) A LOWGA operator of dimension n is a mapping LOWGA: $\bar{S}^n \to \bar{S}$, which has associated with it an exponential weighting vector $w = (w_1, w_2, \ldots, w_n)^T$, with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$, such that

$$\begin{aligned}\text{LOWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \\ &= (s_{\beta_1^{w_1}}) \otimes (s_{\beta_2^{w_2}}) \otimes \cdots \otimes (s_{\beta_n^{w_n}}) = s_\beta,\end{aligned} \tag{2}$$

where $\beta = \prod_{j=1}^n \beta_j^{w_j}$, s_{β_j} is the jth largest element in the set $\{s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\}$.

3. An induced linguistic ordered weighted geometric averaging operator

The LWGA operator weights the linguistic argument itself and the LOWGA operator weights the ordered position of the linguistic argument. That is to say, both the operators consider only one aspect. We shall propose an Induced Linguistic Ordered Weighted Geometric Averaging (ILOWGA) operator that considers not only the linguistic argument itself but also the ordered position of the linguistic argument in the following.

Definition 4. An ILOWGA operator is a mapping ILOWGA: $\bar{S}^n \to \bar{S}$, in which, $w = (w_1, w_2, \ldots, w_n)^T$ is an exponential weighting vector with $w_j \in [0,1]$ and $\sum_{j=1}^n w_j = 1$, $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ is the weighting of the s_{α_i} with $\varpi_j \in [0,1]$ and $\sum_{j=1}^n \varpi_j = 1$ such that

$$\begin{aligned}\text{ILOWGA}_{\varpi,w}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \\ &= (s_{\beta_1^{w_1}}) \otimes (s_{\beta_2^{w_2}}) \otimes \cdots \otimes (s_{\beta_n^{w_n}}) \\ &= s_\beta,\end{aligned} \tag{3}$$

where $\beta = \prod_{j=1}^n \beta_j^{w_j}$, s_{β_j} is the jth largest element in the set $\{\bar{s}_{\alpha_1}, \bar{s}_{\alpha_2}, \ldots, \bar{s}_{\alpha_n}\}$, *i.e.*,

$$\bar{s}_{\alpha_i} = r\varpi_i s_{\alpha_i} = s_{r\varpi_i\alpha_i},$$

in which, $i = 1, 2, \ldots, n$, r is the balancing coefficient, the value of r is usually equal to n.

Example 5. Assume $\varpi = (0.4, 0.3, 0.1, 0.2)^T$, $w = (0.3, 0.2, 0.4, 0.1)^T$, $r = 4$, and

$$s_{\alpha_1} = s_3, s_{\alpha_2} = s_1, s_{\alpha_3} = s_4, s_{\alpha_4} = s_5.$$

According to (3), we have

$$\bar{s}_{\alpha_1} = 4 \times 0.4 \times s_3 = s_{4.8}, \qquad \bar{s}_{\alpha_2} = 4 \times 0.3 \times s_1 = s_{1.2},$$
$$\bar{s}_{\alpha_3} = 4 \times 0.1 \times s_4 = s_{1.6}, \qquad \bar{s}_{\alpha_4} = 4 \times 0.2 \times s_5 = s_{4.0}.$$

Thus, we have

$$s_{\beta_1} = s_{4.8}, \qquad s_{\beta_2} = s_{4.0}, \qquad s_{\beta_3} = s_{1.6}, \qquad s_{\beta_4} = s_{1.2}.$$

Finally, we have

$$\text{ILOWGA}_{\varpi, w}(s_3, s_1, s_4, s_5) = (s_{4.8})^{0.3} \otimes (s_{4.0})^{0.2} \otimes (s_{1.6})^{0.4} \otimes (s_{1.2})^{0.1} = s_{2.6}.$$

The weight vector $w = (w_1, w_2, \dots, w_n)^T$ associated with the ILOWGA operator can be obtained by using the following expression [19,9]:

$$w_j = Q(j/n) - Q\left[(j-1)/n\right], \quad j \in \{1, 2, \dots, n\} \tag{4}$$

where

$$Q(r) = \begin{cases} 0, & r < a, \\ (r-a)/(b-a), & a \leqslant r \leqslant b, \\ 1, & r > b \end{cases} \tag{5}$$

with $a, b, r \in [0, 1]$. Some proportional fuzzy quantifiers such as "most", "at least" and "as many as possible" are usually utilized to get the weight vector $w = (w_1, w_2, \dots, w_n)^T$, where the parameters (a, b) are $(0.3, 0.8)$, $(0, 0.5)$, and $(0.5, 1)$, respectively. Obtaining the attribute weight vector $\varpi = (\varpi_1, \varpi_2, \dots, \varpi_n)^T$ will be discussed in Section 4, which is associated with the index system of coal mine safety evaluation.

Theorem 6 *(Commutativity). Assume f is the ILOWGA operator. Let $(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n})$ be an arbitrary linguistic argument vector, $(r\varpi_1 s'_{\alpha_1}, r\varpi_2 s'_{\alpha_2}, \dots, r\varpi_n s'_{\alpha_n})$ be a permutation of the elements in $(r\varpi_1 s_{\alpha_1}, r\varpi_2 s_{\alpha_2}, \dots, r\varpi_n s_{\alpha_n})$. Then,*

$$f(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = f\left(s'_{\alpha_1}, s'_{\alpha_2}, \dots, s'_{\alpha_n}\right).$$

Proof. For each j, where s_{β_j} is the jth largest element in the set $\{\bar{s}_{\alpha_i}, \bar{s}_{\alpha_2}, \dots, \bar{s}_{\alpha_n}\}$ $(\bar{s}_{\alpha_i} = r\varpi_i s_{\alpha_i},\ i = 1, 2, \dots, n)$ and s'_{β_j} is the jth largest element in the set $\{\bar{s}'_{\alpha_i}, \bar{s}'_{\alpha_2}, \dots, \bar{s}'_{\alpha_n}\}$ $(\bar{s}'_{\alpha_i} = r\varpi_i s'_{\alpha_i},\ i = 1, 2, \dots, n)$, for $(r\varpi_1 s'_{\alpha_1}, r\varpi_2 s'_{\alpha_2}, \dots, r\varpi_n s'_{\alpha_n})$ being a permutation of the elements in $(r\varpi_1 s_{\alpha_1}, r\varpi_2 s_{\alpha_2}, \dots, r\varpi_n s_{\alpha_n})$, we can obtain $s_{\beta_j} = s'_{\beta_j}$ and $(s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \dots \otimes (s_{\beta_n})^{w_n} = (s'_{\beta_1})^{w_1} \otimes (s'_{\beta_2})^{w_2} \otimes \dots \otimes (s'_{\beta_n})^{w_n}$. Hence,

$$f(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = f\left(s'_{\alpha_1}, s'_{\alpha_2}, \dots, s'_{\alpha_n}\right). \qquad \square$$

Theorem 7 *(Idempotency). Assume f is the ILOWGA operator. Let $(s_{\alpha_1}, s_{\alpha_2}, \dots, _{\alpha_n})$ be an arbitrary linguistic argument vector. If $r\varpi_i s_{\alpha_i} = s_\alpha$, for all $i = 1, 2, \dots, n$, then,*

$$f(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) = s_\alpha.$$

Proof. For s_{β_j} is the jth largest element in the set $\{r\varpi_1 s_{\alpha_1}, r\varpi_2 s_{\alpha_2}, \dots, r\varpi_n s_{\alpha_n}\}$ and $r\varpi_i s_{\alpha_i} = s_\alpha$, for all $i = 1, 2, \dots, n$. Then,

$$\begin{aligned} f(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \dots \otimes (s_{\beta_n})^{w_n} \\ &= (s_\alpha)^{w_1} \otimes (s_\alpha)^{w_2} \otimes \dots \otimes (s_\alpha)^{w_n} \\ &= (s_\alpha)^{\sum_{j=1}^n w_j} = s_\alpha. \qquad \square \end{aligned} \tag{6}$$

Theorem 8 *(Monotonicity). Assume f is the ILOWGA operator. Let $(s_{\alpha_1}, s_{\alpha_2}, \dots, _{\alpha_n})$ and $(s'_{\alpha_1}, s'_{\alpha_2}, \dots, s'_{\alpha_n})$ be two arbitrary linguistic argument vectors. For each i, $r\varpi_i s_{\alpha_i} \geqslant r\varpi_i s'_{\alpha_i}$, then,*

$$f(s_{\alpha_1}, s_{\alpha_2}, \dots, s_{\alpha_n}) \geqslant f\left(s'_{\alpha_1}, s'_{\alpha_2}, \dots, s'_{\alpha_n}\right).$$

Proof. For each j, $r\varpi_j s_{\alpha_j} \geqslant r\varpi_j s'_{\alpha_j}$, where s_{β_j} is the jth largest element in the set $\{r\varpi_1 s_{\alpha_1}, r\varpi_2 s_{\alpha_2}, \ldots, r\varpi_n s_{\alpha_n}\}$ and s'_{β_j} is the jth largest element in the set $\{r\varpi_1 s'_{\alpha_1}, r\varpi_2 s'_{\alpha_2}, \ldots, r\varpi_n s'_{\alpha_n}\}$, so $(s_{\beta_j})^{w_j} \geqslant (s'_{\beta_j})^{w_j}$ and $(s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \geqslant (s'_{\beta_1})^{w_1} \otimes (s'_{\beta_2})^{w_2} \otimes \cdots \otimes (s'_{\beta_n})^{w_n}$. For $f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n}$ and $f(s'_{\alpha_1}, s'_{\alpha_2}, \ldots, s'_{\alpha_n}) = (s'_{\beta_1})^{w_1} \otimes (s'_{\beta_2})^{w_2} \otimes \cdots \otimes (s'_{\beta_n})^{w_n}$, we can obtain $f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) \geqslant f(s'_{\alpha_1}, s'_{\alpha_2}, \ldots, s'_{\alpha_n})$. □

Theorem 9 *(Bounded). Assume f is the ILOWGA operator. Then,*

$$Min(s_{\beta_i}) \leqslant f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) \leqslant Max(s_{\beta_i}).$$

Proof. Let $Max(s_{\beta_i}) = s_\beta$ and $Min(s_{\beta_i}) = s_\alpha$. Then,

$$\begin{aligned} f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \\ &\leqslant (s_\beta)^{w_1} \otimes (s_\beta)^{w_2} \otimes \cdots \otimes (s_\beta)^{w_n} = (s_\beta)^{\sum_{j=1}^n w_j} = s_\beta; \\ f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \\ &\geqslant (s_\alpha)^{w_1} \otimes (s_\alpha)^{w_2} \otimes \cdots \otimes (s_\alpha)^{w_n} = (s_\alpha)^{\sum_{j=1}^n w_j} = s_\alpha. \end{aligned}$$

Hence,

$$Min(s_{\beta_i}) \leqslant f(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) \leqslant Max(s_{\beta_i}). \qquad \square$$

Theorem 10. *If $\varpi = (1/r, 1/r, \ldots, 1/r)^T$, then*

$$\mathrm{ILOWGA}_{\varpi, w}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \mathrm{LOWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}). \tag{7}$$

Proof. By $\varpi = (\frac{1}{r}, \frac{1}{r}, \ldots, \frac{1}{r})^T$, then $r\varpi_i s_{\alpha_i} = r\frac{1}{r} s_{\alpha_i} = s_{\alpha_i}$, so, s_{β_j} is the jth largest element in the set $\{\bar{s}_{\alpha_1}, \bar{s}_{\alpha_2}, \ldots, \bar{s}_{\alpha_n}\}$ ($\bar{s}_{\alpha_i} = s_{r\varpi_i \alpha_i}$, $i = 1, 2, \ldots, n$), *i.e.*, s_{β_j} is the jth largest element in the set $\{s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}\}$ ($\bar{s}_{\alpha_i} = s_{\alpha_i}$, $i = 1, 2, \ldots, n$). Hence,

$$\begin{aligned} \mathrm{ILOWGA}_{\varpi, w}(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) &= (s_{\beta_1})^{w_1} \otimes (s_{\beta_2})^{w_2} \otimes \cdots \otimes (s_{\beta_n})^{w_n} \\ &= (s_{\beta_1^{w_1}}) \otimes (s_{\beta_2^{w_2}}) \otimes \cdots \otimes (s_{\beta_n^{w_n}}) \\ &= \mathrm{LOWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}). \qquad \square \end{aligned}$$

4. A decision-making method for coal mine safety evaluation

Safety, an important subject concerned by all nations, is not only an eternal topic in coal mining but also is fundamental in the process of coal mine production. Therefore, safety is one of key issues in coal mining that needs to be addressed at the highest level. The safety problem occurs because of the underground working conditions of coal mines which involve many hazards, such as mesh gas, coal dust, unstable wall rocks, spontaneous fire of coal seams and flooding. Therefore, it calls for an urgent implementation of a method of safety evaluation in the process of production by the technology and management departments of a coal mine, so that potential risks to personnel may be predicted and relevant measures can be taken to prevent accidents and to achieve safety in production with a minimum investment.

4.1. Establishing a safety evaluation indicator system

A coal mine safety evaluation deals with the safety of the entire production system of a coal mine. The safety indices comprise an entity that is composed of interactive and interconnected factors relating to the safe production of a coal mine. These safety indices form the basis of the evaluation. Whether the selection of the evaluation indicators is reasonable or not will affect the integrated evaluation results, so it is important to establish a scientifically justified evaluation system. In this paper, we defined the safety of a coal mine as a general target hierarchy, with six main factors as the rule hierarchy which affects the safety of a coal mine. First we carried out a qualitative analysis, with the following evaluation indicators, listed in Fig. 1.

Six main factors of coal mine safety evaluation are usually evaluated by the experts with six linguistic term sets, as shown in Table 1.

4.2. Obtain the attribute (factor) weight vector

The attribute weight vector $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$ is obtained by the following procedures. By establishing the estimate matrix pairwise, we can show the relative importance between some hierarchical factors and that of its upper hierarchy from the matrix of grades designed by the experts. The matrix $B_{n\times n}$ represents the security status of a coal mine, graded

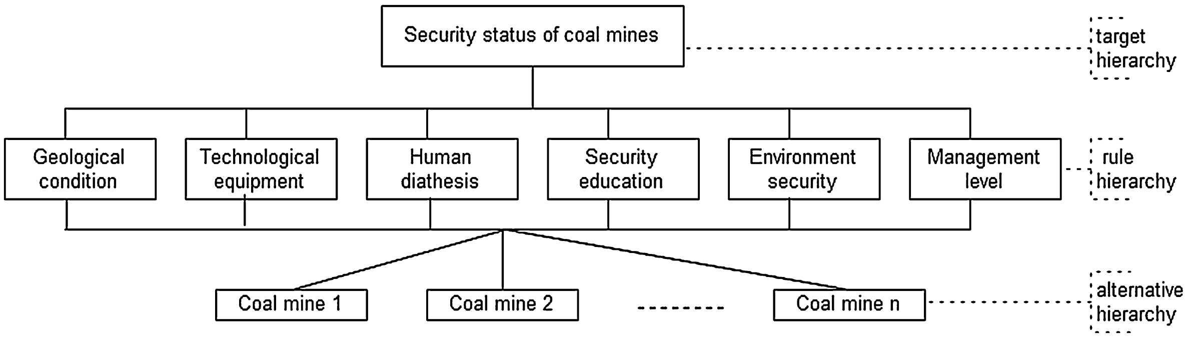

Fig. 1. Index system of coal mine safety evaluation.

Table 1
The linguistic term sets of the factors.

Factors	Linguistic term sets S_i $(i = 1, 2, \ldots, 6)$
geological condition	{very active, active, fair, steady, very steady}
technological equipment	{very behindhand, behindhand, fair, advanced, very advanced}
human diathesis	{very low, low, fair, high, very high}
security education	{very poor, poor, fair, good, very good}
environment security	{very formidable, formidable, fair, fine, very fine}
management level	{very low, low, fair, high, very high}

by experts as follows: the numbers 1 to 9 or their reciprocals are the gradations of importance, where the number 1 represents two factors of equal importance and the numbers 3, 5, 7 and 9 represent the following qualitative evaluations: rather important, apparently important, intensively important and especially important respectively. The numbers 2, 4, 6 and 8 represent the intermediate numbers of 1, 3, 5, 7 and 9 and the reciprocals of the previous numbers stand for the degree of important of the other set compared with the former when they are compared pairwise. We obtained the weight matrix $B_{n\times n}$ of the rule hierarchy to the total target as the following matrix according to the established index system of the security evaluation of a coal mine based on some research results of scholars at home and abroad and the advices of the specialists in the study of the coal mine safety:

$$B = \begin{bmatrix} b_{11} & b_{12} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{n1} & b_{n2} & \cdots & b_{nn} \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 & 5 & 3 & 4 \\ 1/2 & 1 & 2 & 4 & 2 & 3 \\ 1/3 & 1/2 & 1 & 2 & 1/2 & 1/2 \\ 1/5 & 1/4 & 1/2 & 1 & 1/4 & 1/3 \\ 1/3 & 1/2 & 2 & 4 & 1 & 2 \\ 1/4 & 1/3 & 2 & 3 & 1/2 & 1 \end{bmatrix} \tag{8}$$

where $b_{ii} = 1$, $b_{ij} \cdot b_{ji} = 1$, $b_{ij} > 0$; $\forall i, j \in \{1, 2, \ldots, n\}$.

Compute the vector of weights. Common methods of computing the sequencing of factor weights are, among others, the following: ANC, square root algorithm, eigenvalue method and the least squares method. We adopted the square root algorithm to compute the weight vector $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$. Its procedure is as follows:

a) After computing the geometric mean of all factors in every row of the estimated $B_{6\times 6}$ matrix: we obtained the vector $m = (m_1, m_2, \ldots, m_6)^T$:

$$m_i = \sqrt[6]{\prod_{j=1}^{6} b_{ij}} \quad \bigl(i, j \in \{1, 2, \ldots, 6\}\bigr). \tag{9}$$

By using (9), we obtain the vector $m = (2.667, 1.698, 0.661, 0.357, 1.178, 0.794)^T$.

b) From the normalized column vector m, we obtain the attribute weight vector $\varpi = (\varpi_1, \varpi_2, \ldots, \varpi_n)^T$:

$$\varpi_i = \frac{m_i}{\sum_{j=1}^{6} m_j} \quad \bigl(i, j \in \{1, 2, \ldots, 6\}\bigr). \tag{10}$$

By using (10), we obtain the attribute vector $\varpi = (0.363, 0.231, 0.090, 0.049, 0.160, 0.107)^T$.

Verification of the estimation matrix for consistency. The sequenced weight vector needed to be verified for consistency, because the experts may have been subjective and biased in the process of establishing the estimation matrix, given the complexity of being objective. The procedure for verification of consistency of the estimation matrix is as follows: first, we

Table 2
RI values of mean consistency index.

n	1	2	3	4	5	6	7	8	9	10	11	12
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46	1.49	1.52	1.54

Table 3
The decision information about coal mines safety.

	u_1	u_2	u_3	u_4	u_5	u_6
x_1	fair	behindhand	low	very good	fair	fair
x_2	steady	fair	fair	poor	good	fair
x_3	very steady	fair	high	good	good	fair
x_4	very steady	fair	very high	good	fine	very high
x_5	very steady	very advanced	high	good	very fine	fair

obtain the i component of the column vector $B\varpi$ given the estimation matrix $B_{6\times 6}$ post multiplied by the attribute weight vector ϖ. We obtain the maximum characteristic root of $B_{6\times 6}$:

$$\lambda_{\max} = \sum_{i=1}^{6} \frac{(B\varpi)_i}{6\varpi_i}. \tag{11}$$

We use the following formula for consistency verification: $CR = CI/RI$, where CR is the consistency proportion and CI the consistency index, computed as follows: $CI = (\lambda_{\max} - 6)/(6-1)$; RI is the mean max consistency index, the data is shown in Table 1. When $CR < 0.1$, we can accept that the estimation matrix has satisfied consistency, otherwise we must adjust the factors of the estimation matrix until it has satisfied consistency. We calculated the maximum characteristic root $\lambda_{\max}$ as 6.21 and CI as 0.042. When $n = 6$, RI is 1.26 (listed in Table 2) and CR is 0.033, less than 0.1, which certified that the estimation matrix pairwise satisfied consistency.

4.3. Application of the decision-making method

In this section, a decision-making problem involves the evaluation of five coal mines safety x_i $(i = 1, 2, 3, 4, 5)$ of an area. Making a decision, the attributes considered include: u_1: geological condition; u_2: technological equipment; u_3: human diathesis; u_4: security education; u_5: environment security; u_6: management level. The decision information about coal mines safety is presented in Table 3.

To get the safest mine(s), the following steps are involved:

Step 1 According to Table 3, the linguistic decision matrix A for the decision making for coal mine safety is

$$A = \begin{bmatrix} s_3 & s_2 & s_2 & s_5 & s_3 & s_3 \\ s_4 & s_3 & s_3 & s_2 & s_4 & s_3 \\ s_5 & s_3 & s_4 & s_4 & s_4 & s_3 \\ s_5 & s_3 & s_5 & s_4 & s_4 & s_5 \\ s_5 & s_5 & s_4 & s_4 & s_5 & s_3 \end{bmatrix}.$$

Step 2 Utilize the fuzzy linguistic quantifier "most" with the pair $(0.3, 0.8)$, and by (4) and (5), we obtain the weighting vector (associated with the ILOWGA operator) $w = (0, 0.067, 0.333, 0.333, 0.267, 0)^T$. In addition, we have obtained the attribute vector $\varpi = (0.363, 0.231, 0.090, 0.049, 0.160, 0.107)^T$ in Section 4.2.

Step 3 Utilize the decision information given in matrix A and the ILOWGA operator

$$z_i = \mathrm{ILOWGA}_{\varpi,w}(s_{\alpha_{i1}}, s_{\alpha_{i2}}, \ldots, s_{\alpha_{in}}). \tag{12}$$

According to (3), we have

$$\bar{s}_{\alpha_1} = 6 \times 0.363 \times s_3 = s_{6\times 0.363\times 3} = s_{6.534},$$

$$\bar{s}_{\alpha_2} = 6 \times 0.231 \times s_2 = s_{6\times 0.231\times 2} = s_{2.772},$$

$$\bar{s}_{\alpha_3} = 6 \times 0.090 \times s_2 = s_{6\times 0.090\times 2} = s_{1.08},$$

$$\bar{s}_{\alpha_4} = 6 \times 0.049 \times s_5 = s_{6\times 0.049\times 5} = s_{1.47},$$

$$\bar{s}_{\alpha_5} = 6 \times 0.160 \times s_3 = s_{6\times 0.160\times 3} = s_{2.88},$$

$$\bar{s}_{\alpha_6} = 6 \times 0.107 \times s_3 = s_{6\times 0.107\times 3} = s_{1.926}.$$

Thus, we have

$$s_{\beta_1} = s_{6.534}, \quad s_{\beta_2} = s_{2.88}, \quad s_{\beta_3} = s_{2.772}, \quad s_{\beta_4} = s_{1.926}, \quad s_{\beta_5} = s_{1.47}, \quad s_{\beta_6} = s_{1.08},$$

Table 4
The comparative results of different operators.

Operators	Results of ranking the alternative
LWGA	$z_5 \succ z_4 \succ z_3 \succ z_1 \succ z_2$
LOWGA	$z_4 = z_5 \succ z_3 \succ z_2 \succ z_1$
ILOWGA	$z_4 \succ z_5 \succ z_3 \succ z_2 \succ z_1$

and the overall value z_i of alternative x_i,

$$\begin{aligned}
z_1 &= \mathrm{ILOWGA}_{\varpi,w}(s_{\alpha_{11}}, s_{\alpha_{12}}, \ldots, s_{\alpha_{1n}}) = \mathrm{ILOWGA}_{\varpi,w}(s_3, s_2, s_2, s_5, s_3, s_3) \\
&= (s_{\beta_1})^0 \otimes (s_{\beta_2})^{0.067} \otimes (s_{\beta_3})^{0.333} \otimes (s_{\beta_4})^{0.333} \otimes (s_{\beta_5})^{0.267} \otimes (s_{\beta_6})^0 \\
&= (s_{6.534})^0 \otimes (s_{2.88})^{0.067} \otimes (s_{2.772})^{0.333} \otimes (s_{1.926})^{0.333} \otimes (s_{1.47})^{0.267} \otimes (s_{1.08})^0 \\
&= s_{(6.534^0 \times 2.88^{0.067} \times 2.772^{0.333} \times 1.926^{0.333} \times 1.47^{0.267} \times 1.08^0)} \\
&= s_{2.0782}.
\end{aligned}$$

Finally, we obtain the other values through the same method:

$$z_2 = s_{2.4366}, \qquad z_3 = s_{2.6511}, \qquad z_4 = s_{3.3105}, \qquad z_5 = s_{2.9550}.$$

Step 4 Utilize z_i $(i = 1, 2, 3, 4)$ to rank the alternative as follows:

$$z_4 \succ z_5 \succ z_3 \succ z_2 \succ z_1.$$

Thus the best safety coal mine is x_4.

4.4. Discussion on the different methods

In this section, we'll replace the above proposed method with the traditional method based on LWGA operator. *Step 1*, *Step 2* and *Step 4* are the same with the above proposed method. To get the safest mine(s), *Step 3* involved is as follows:

Utilize the decision information given in matrix A and the LWGA operator

$$z_i = \mathrm{LWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}). \tag{13}$$

According to (1), we can obtain the overall value z_i of alternative x_i,

$$\begin{aligned}
z_1 &= \mathrm{LWGA}_w(s_{\alpha_1}, s_{\alpha_2}, \ldots, s_{\alpha_n}) = \mathrm{LWGA}_w(s_3, s_2, s_2, s_5, s_3, s_3) \\
&= (s_3)^0 \otimes (s_2)^{0.067} \otimes (s_2)^{0.333} \otimes (s_5)^{0.333} \otimes (s_3)^{0.267} \otimes (s_3)^0 \\
&= s_{(3^0 \times 2^{0.067} \times 2^{0.333} \times 5^{0.333} \times 3^{0.267} \times 3^0)} \\
&= s_{3.0239}.
\end{aligned}$$

Finally, we obtain the other values through the same method:

$$z_2 = s_{2.8304}, \qquad z_3 = s_{3.9236}, \qquad z_4 = s_{4.2263}, \qquad z_5 = s_{4.3095}.$$

Utilize z_i $(i = 1, 2, 3, 4)$ to rank the alternative as follows:

$$z_5 \succ z_4 \succ z_3 \succ z_1 \succ z_2.$$

We can obtain the rank of the alternative as follows, if we replace the LWGA operator with the LOWGA operator:

$$z_4 \succ z_5 \succ z_3 \succ z_2 \succ z_1.$$

The comparative results of different methods are presented in Table 4. According to Table 4, we can perceive the remarkable difference in the ordered results obtained through LWGA operator and through LOWGA operator and ILOWGA operator. Because LWGA operator doesn't weight the ordered position of the linguistic argument, that's to say the max attribute and the min attribute should have been elided. We can see $z_4 = z_5$ through LOWGA operator and $z_4 \succ z_5$ through ILOWGA, because LOWGA operator only weights the ordered position of the linguistic argument but it doesn't weight the linguistic argument itself. However, the ILOWGA operator not only considers the weight of the factors affecting coal mine safety, but also takes the ordered position of the factors in aggregation process.

5. Conclusions

In this paper, we developed an ILOWGA operator, in which both the given of linguistic argument itself and the ordered position of linguistic argument have been considered. That is to say, we not only consider the weight of the attribute but also consider the weight associated with the ILOWGA operator in aggregating decision information. And then, we studied the index system of coal mine safety evaluation and proposed a decision-making method for coal mine safety evaluation with linguistic values. The theoretical analysis and the comparative results show that the decision making based on ILOWGA operator can better reflect the real situations in coal mine safety evaluation.

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